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AN EXAMPLE OF TRANSONIC FLOW PAST A BODY WITH DISCONTINUITY IN THE BODY PROFILE CURVATURE*

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The exact solution of transonic equations, which is an extension of the known self-similar solution that defines transonic flow around a corner point of the body contour is investigated.

The supersonic flow around blunt bodies with a break or curvature discontinuity in their profiles were investigated theoretically and numerically in /1,2/. Typical shapes of bodies, of the shock wave and sonic line are diagrammatically shown in Fig.1 in the case of contour break (a) and curvature discontinuity (b). It was shown that the local flows around the corner is defined by the Karman—Fal'kovich transonic equation

$$uu_x = v_y, \quad u_y = v_x \tag{1.1}$$

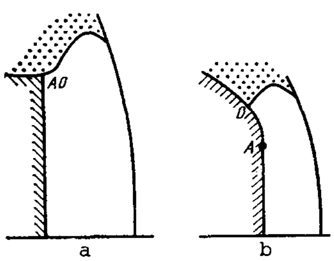

Fig.1

where u and v are normalized dimensionless perturbation velocity oomponents in a uniform sonic flow, and x and y are Cartesian coordinates of a local system.

Using the hodograph representation it is possible to write the solution of system (1), that is valid in the neighborhood of point AO, in the form

$$x = {}^4/_5\,({}^3/_2)^{1/3} K\,[(\rho - v)^{2/3}\,(3v + 2\rho) + (\rho + v)^{2/3}\,(2\rho - 3v)] \tag{2}$$

$$y = K\,[(\rho + v)^{1/3}\,(3v - \rho) + (\rho - v)^{1/3}\,(3v + \rho)]\,, \quad (\rho = (v^2 - {}^4/_9 u^3)^{1/2})$$

A method was developed in /3/ for obtaining new solutions of transonic equations, which consists of the substitution of the complex argument $V = v + i\lambda$ (λ is the real variable for the variable v in the hodograph solution. Subsequent division of the result by the real and imaginary parts yield two new solutions of the transonic system. We apply this idea to (2) and obtain a new solution which, similarly to (2), satisfies the symmetry condition ($y = 0$ when $v = 0$) at the subsonic part of the body contour ($u < 0$)

$$x_1 = \operatorname{Re} x\,(u,\, V); \quad y_1 = \operatorname{Re} y\,(u,\, V) \tag{3}$$

Using the parametric form we can represent it in terms of real quantities. Some detail of such parametrization appeared in /4/. Denoting the two real parameters by p and q, we obtain for system (1) the solution

$$u = \frac{C_1\,(1 - C_2 p^3)}{2p^2} - \frac{C_1^2 q^2}{4}, \quad v = C_1^2 q\left(\frac{2 + C_2 p^3}{4p^2} - \frac{C_1 q^3}{12}\right) \tag{4}$$

$$x = C_1^3 q^2\,\frac{(4 - C_2 p^3)}{16 p^3} + \frac{C_1^2}{40 p^5}\,(10 C_2^2 p^6 - 5 C_2 p^3 + 4)$$

$$y = C_1^2 q\,\frac{(2 + C_2 p^3)}{4 p^3}$$

which is equivalent to solution (3). In this equation C_1 and C_2 are constants that are related to K and λ appearing in (3) by formulas

$$\lambda = -C_2(-C_1 / 2)^{3/2} 3^{1/2}, \quad K = 2^{23/3} 3^{4/3} 5^{-5}$$

When $C_2 = 0$, solution (4) defines the self-similar solution

$$\tau = -\frac{125}{48} + \frac{5}{2}\,t \pm \left(1 - \frac{16}{25}\,t\right)^{3/2}, \quad \left(t = u\left(\frac{y}{x}\right)^2, \quad \tau = v\left(\frac{y}{x}\right)^3\right)$$

obtained in /5/ for the flow considered in /1/. The sonic line and all isolines $u = \text{const} > 0$, $v = \text{const} > 0$ converge at the single point AO, as shown in Fig.1,b, which corresponds to infinite values of parameter p in (4).

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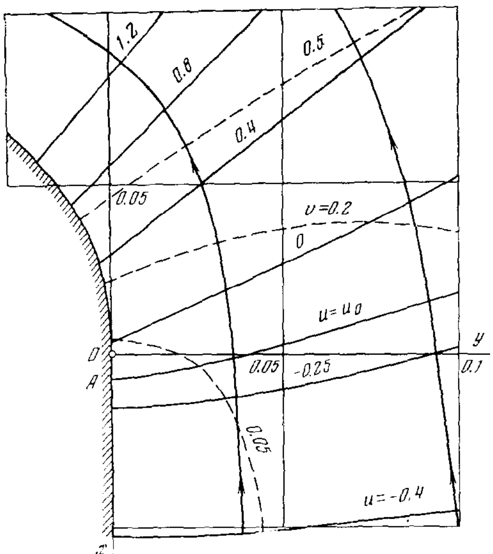

Fig.2

The velocity field and the streamlines obtaining near a straight wall joined at point A to some line of finite curvature are shown in Fig.2. Calculations were carried out for $C_1 = -2^{-7}\,3^{-3}\,5^5$, $C_2 = -0.09496$. To indicate the region of parameter variation we determine the images of axes $x = 0$ and $y = 0$ in the pq-plane. The part of axis $q = 0$ from $p = 0$ to $p_0 = -(2/C_2)^{1/3}$ corresponds to axis $y = 0$ between $x = +\infty$ and $x_A > 0$ when condition $v = 0$ is satisfied. The velocity at that point is characteristic for a given set of C_1 and C_2

$$x_A = \frac{27}{20} C_1^2 (-C_2/2)^{5/3} \approx 0.0069$$

$$u_A = 1.5 C_1 (-C_2/2)^{2/3} \approx -0.1778$$

Along the given part of axis $x > 0$ the velocity is throughout subsonic increasing to u_A at point A. Formula (4) implies that the straight line $C_2 p_0^3 = -2$ lies along the remaining part of the axis $y = 0$ between point A and the coordinate origin $x = 0$ and then continues along the negative semiaxis x. Along that part of the axis $v \neq 0$ and velocity u increases from transonic u_A to $+\infty$ $(q \to \infty)$. At the coordinate origin $x = y = 0$ we have $p = p_0$, $q = (18/5)^{1/2} C_1^{-1/2} (-C_2/2)^{1/3}$.

The streamlines and part of the wall beyond point A were obtained by integrating the equation $dy/dx = v$.

The isoline $u = \text{const} > 0$ fairly quickly becomes rectilinear, as indicated in Fig.2. This illustrates one of the properties of the obtained solution which asymptotically becomes the Prandtl—Meyer rarefaction wave. We would recall that this property played an important part in the determination of transonic flow over a corner.

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